NATURAL CONVECTION IN A VERTICAL CHANNEL IN A NONUNIFORM MAGNETIC FIELD

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It is shown that a nonuniform magnetic field suppresses the natural convection and shapes the specific MHD flow.

The study of the effect of a magnetic field on the structure of flow and convective heat exchange during natural convection attracts the interest of many investigators. This problem has been taken up in the literature. However, the studies were conducted for the most part for semiinfinite geometries and uniform magnetic fields. Information on these studies can be found in [1-5]. In this article we are interested in natural convection in channels and closed cavities.

A theory on the MHD effect of free thermal convection of an electrically conducting liquid in a vertical round pipe in a weak magnetic field was presented by Smirnov [6]. He wrote a number of articles on a theoretical and experimental study of thermal convection of mercury in a closed pipe in a transverse magnetic field [7-8]. Gershuni and Zhukhovitskii [9-10] studied stationary convective movement of an electrically conducting liquid between vertical parallel plates in a transverse uniform magnetic field. In the articles the distribution of velocity, temperature, and induced magnetic field was determined, and the convective heat flux was calculated. It was shown that with an increase in the field the convective flow retains its symmetry but is rapidly retarded. This problem was generalized by Regirer [11] for the case of the presence of a vertical temperature gradient. It was shown that the presence of a magnetic field considerably delays the onset of instability in the equilibrium. A solution of the MHD problem of convection generated in an electrically conducting liquid in vertical channels can be found in the articles of Regirer [12-13].

Plane stationary convective MHD movement in a rectangular cavity in the presence of a horizontal magnetic field was studied by Singh and Cowling [14].

Agarwal [15] investigated natural convection in a horizontal channel with porous walls in a uniform transverse magnetic field. The liquid was forced in through the lower wall of the channel and was drawn off through the upper wall. It was shown that the magnetic field leads to a decrease in the level of velocities and temperatures in the channel and to a decrease in the Nusselt number.

The study of the effect of a nonuniform magnetic field on natural convection is timely at present. This problem was examined for a semiinfinite geometry in a number of articles [16-18], on which we will not dwell. The free convection of an electrically conducting liquid inside a closed surface, located in a nonuniform transverse magnetic field, was studied by Emery [19]. It was shown that if the velocity, temperature, thickness of the boundary layer, and magnetic induction vary in a gradual way: $v \sim Ay^n$; $\theta_W - \theta_\infty \sim cy^\gamma$; $\delta \sim Ny^m$, and $B \sim Dy^\beta$, then a solution exists in two cases: 1) $n = 1/2$, $\gamma = 0$, $m = 1/4$, $\beta = -1/4$ and 2) $n = 1$, $\gamma = 1$, $m = 0$, $\beta = 0$. For both cases equations are obtained for the ratio of the Nusselt number in the presence of a magnetic field N_M and without a magnetic field N. The ratio N_M/N depends on the Prandtl number and the parameter $B^2/\sqrt{\Delta\theta}$. The ratio N_M/N decreases with an increase in $B^2/\sqrt{\Delta\theta}$. The theoretical calculations were qualitatively confirmed by the results of experiments.

Convective instability of a conducting liquid in a long vertical pipe of circular cross section was studied by Pisarev [20]. The magnetic field was created by a current flowing through the liquid parallel to

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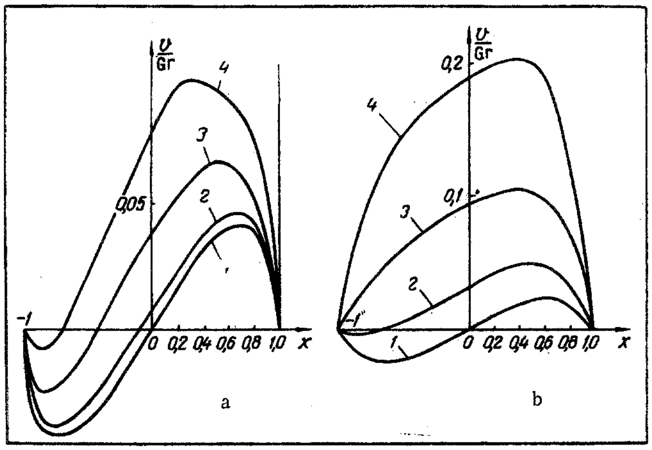

Fig. 1. Dependence of velocity profile on γ at M = 2 (a) and at M = 4 (b): 1) γ = 0; 2) 0.01; 3) 0.05; 4) 0.1.

the gravitational field. The vertical temperature gradient examined was also parallel to the gravitational field. The dependence of the critical Rayleigh number on the Hartman number was obtained in the article. A disruption in the mechanical stability of an unevenly heated conducting liquid in a magnetic field takes place on reaching higher values of the critical temperature gradient than in the case of free thermal convection. It was found that, at the ultimate value of the Hartman number, convection may be completely suppressed. It must be noted that there are several inaccuracies in the formulation of the problem. The development of natural convection in an electrically conducting liquid may be connected with a transition through a stage of neutral oscillations. Therefore the Rayleigh number found is not necessarily the smallest. If the magnetic field is nonuniform ($H_\varphi \sim r$) then the boundary conditions for the temperature and the induced magnetic field must be interpreted in some way.

We study the effect of a nonuniform magnetic field on natural convection in a vertical channel. We set down a dimensionless system of equations describing free convection in electrically conducting liquids. The distance between the vertical walls of the channel is equal to $2a$, while the constant temperature difference between the surfaces is $2\theta_1$. We take a and θ_1 as the units of distance and temperature, and ν/a, a^2/ν, and $\rho\nu^2/a^2$ as units of velocity, time, and pressure, respectively.

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\nabla\left(p + \frac{H^2}{8\pi}\right) + \Delta\mathbf{v} + \mathrm{Gr}\,\theta\mathbf{j} + \frac{\mathrm{M}^2}{\mathrm{R}_m}(\mathbf{H}\nabla)\mathbf{H},$$

$$\frac{\partial\theta}{\partial t} + \mathbf{v}\nabla\theta = \frac{1}{\mathrm{Pr}}\Delta\theta, \tag{1}$$

$$\frac{\partial\mathbf{H}}{\partial t} + (\mathbf{v}\nabla)\mathbf{H} - (\mathbf{H}\nabla)\mathbf{v} = \frac{1}{\mathrm{R}_m}\Delta\mathbf{H},$$

$$\operatorname{div}\mathbf{H} = 0, \ \operatorname{div}\mathbf{v} = 0,$$

where $\mathrm{Pr} = \nu/\varkappa$ is the Prandtl number; $\mathrm{M} = \mu H_{0x} a\sqrt{\sigma/\eta}$ is the Hartman number; $\eta = \nu\rho$; $\mathrm{R_m} = \nu\mu\sigma$ is the magnetic Reynolds number; $\mathrm{Gr} = \beta g\theta_1 a^3/\nu^2$ is the Grashof number.

We examine the stationary convection of an electrically conducting liquid in the space between vertical parallel surfaces heated to different temperatures, in an external nonuniform magnetic field. Let the linear dimensions of the plates be many times larger than the separation between the surfaces, so that the problem becomes one-dimensional.

In accordance with the assumptions made, system (1) takes the form:

$$0 = \frac{d^2v}{dx^2} + \mathrm{Gr}\,\theta + \frac{\mathrm{M}^2}{\mathrm{R}_m}H_x\frac{dH_y}{dx}, \quad 0 = \frac{d}{dx}\left(p + \frac{H^2}{8\pi}\right),$$

$$0 = H_x\frac{dv}{dx} + \frac{1}{\mathrm{R}_m}\frac{d^2H_y}{dx^2}, \tag{2}$$

$$0 = \frac{d^2\theta}{dx^2}, \quad H_y = H_y(x), \quad H_x = 1 = \mathrm{const}.$$

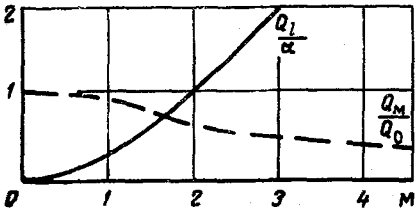

Fig. 2. Dependence of flow rate of liquid Q_l and convective heat flux Q_M on M.

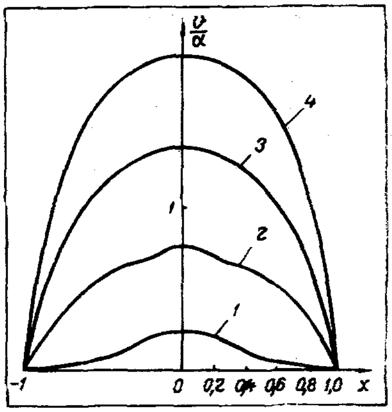

Fig. 3. Dependence of velocity profile on M at Gr = 0: 1) M = 1; 2) 2; 3) 3; 4) 4.

We set the boundary conditions for system (2) in the following form:

$$v = 0, \quad x = \pm 1,$$
$$H_y = H_1, \quad \theta = +1, \quad x = +1, \tag{3}$$
$$H_y = H_2, \quad \theta = -1, \quad x = -1.$$

From system (2) and the boundary conditions (3) we obtain the following solution:

$$v = \frac{\mathrm{Gr}\,(x\,\mathrm{sh}\,\mathrm{M} - \mathrm{sh}\,\mathrm{M}x)}{\mathrm{M}^2\,\mathrm{sh}\,\mathrm{M}} + \frac{\Delta H_y}{\mathrm{R}_m}\,\frac{\mathrm{M}}{2\,\mathrm{sh}\,\mathrm{M}}(\mathrm{ch}\,\mathrm{M} - \mathrm{ch}\,\mathrm{M}x), \tag{4}$$

$$H_y = \frac{H_1 + H_2}{2} + \frac{\Delta H_y\,\mathrm{sh}\,\mathrm{M}x}{2\,\mathrm{sh}\,\mathrm{M}}$$
$$-\frac{\mathrm{Gr}\,\mathrm{R}_m}{\mathrm{M}^3\,\mathrm{sh}\,\mathrm{M}}\left(\mathrm{ch}\,\mathrm{M} - \mathrm{ch}\,\mathrm{M}x\right) - \frac{\mathrm{Gr}\,\mathrm{R}_m}{2\mathrm{M}^2}(x^2 - 1). \tag{5}$$

$$H_x = 1, \quad \theta = x, \quad p = \mathrm{const} - \frac{1 + H_y^2}{8\pi},$$

where $\Delta H_y = H_1 - H_2$.

Let us consider the solution obtained. The first component in the expression for the velocity corresponds to the velocity of convective flow in a vertical infinite slot in an external uniform transverse magnetic field (4), while the second component is due to the nonuniformity of the magnetic field. The total field H_y in Eq. (5) represents the superposition of the induced magnetic field and the component of the external magnetic field. In order to isolate the component of the induced magnetic field we proceed as follows. We assume that the liquid in the channel is not electrically conducting, so that M = 0 and $R_m = 0$ and the induced magnetic field is absent. The equation for determining the y-component of the magnetic field takes the form:

$$\frac{d^2 H_y}{dx^2} = 0.$$

In accordance with the boundary conditions (3) we obtain

$$H_y = \frac{H_1 + H_2}{2} + \frac{\Delta H_y}{2}x.$$

We will assume that this is actually the y-component of the external magnetic field, so that the external field in the general form can be written with the equation

$$\mathbf{H} = 1 \cdot \mathbf{i} + \left(\frac{H_1 + H_2}{2} + \frac{\Delta H_y}{2}x\right)\mathbf{j},$$

where **i** and **j** are unit vectors. In weak magnetic fields where $M \ll 1$ the velocity of convective flow depends weakly on the Hartman number:

$$v = \mathrm{Gr}\,\frac{x - x^3}{6} + \mathrm{Gr}\,\frac{3x^3 + 7x - 10x^5}{12}\,\mathrm{M}^2 + \frac{\Delta H_y}{2\mathrm{R}_m}(1 - x^2)\,\mathrm{M}^2.$$

In strong fields when $M \gg 1$

$$v = \mathrm{Gr}\,\frac{x - x^5}{20 + 21\mathrm{M}^2} + \frac{\Delta H_y}{2\mathrm{R}_m}(1 - x^4)\,\mathrm{M}.$$

We now follow how the nonuniformity of the field affects natural convection. For this we construct the dependence of the velocity on the coordinate x in the form:

$$\frac{v}{\mathrm{Gr}} = \frac{x\,\mathrm{sh}\,\mathrm{M} - \mathrm{sh}\,\mathrm{M}x}{\mathrm{M}^2\,\mathrm{sh}\,\mathrm{M}} + \gamma\,\frac{\mathrm{M}}{2\,\mathrm{sh}\,\mathrm{M}}(\mathrm{ch}\,\mathrm{M} - \mathrm{ch}\,\mathrm{M}x), \quad \gamma = \frac{\Delta H_y}{\mathrm{Gr}\mathrm{R}_m}.$$

The effect of a nonuniform field γ on the process of natural convection is reflected in Fig. 1a, b. The dependence v(x) corresponding to $\gamma = 0$ represents an antisymmetric curve. At some γ different from zero the symmetry is disrupted and some flowing of liquid appears: the upward flow of liquid is greater than the downward flow. The flow rate depends on the value M:

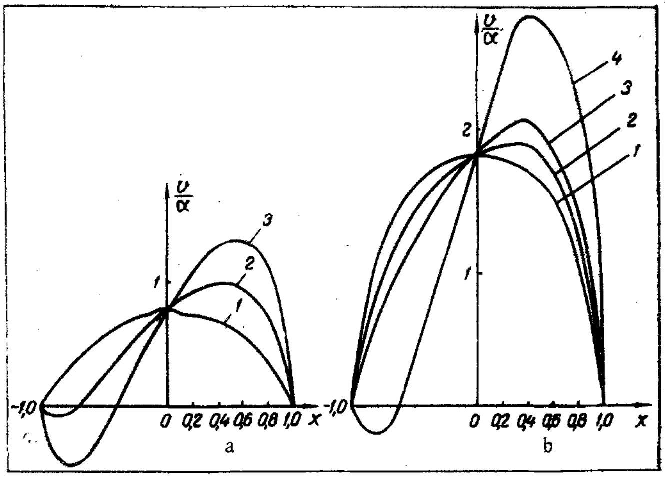

Fig. 4. Dependence of velocity profile on β at M = 2 (a) and at M = 4 (b): 1) β = 0; 2) 10); 3) 20); 4) 60.

$$\frac{Q_l}{\alpha} = M\frac{\operatorname{ch} M}{\operatorname{sh} M} - 1, \tag{6}$$

where $\alpha = \Delta H_y/Rm$; Q_l is the dimensionless flow rate of the liquid.

It is interesting to note that ΔH_y and consequently α were considered to be positive up to now. And in this case the flow of liquid arising under the effect of an external magnetic field is directed vertically upward. With a change in the direction of the magnetic field gradient ΔH_y the flow also changes to the opposite direction.

The dependence of the dimensionless flow rate of liquid on the Hartman number is reflected in Fig. 2. For M > 4 the ratio $\operatorname{ch} M/\operatorname{sh} M$ is practically equal to 1, and therefore in the region M > 4

$$\frac{Q_l}{\alpha} \simeq M - 1, \tag{7}$$

i.e., in the region of large M the flow rate of liquid is a linear function of the Hartman number. It is seen from a comparison of Fig. 1a, b that for a given field nonuniformity γ its effect strengthens with an increase in the Hartman number. Thus, for γ = 0.05 at M = 2 the downward flow is large, while for the same γ and M = 4 this flow is generally absent. The explanation is evidently that with an increase in M the natural convection is more strongly damped, while the MHD flow produced by the field nonuniformity becomes more intense with an increase in M.

This problem can be examined from another side: tracing the effect of free convection on the development of MHD flow under the effect of a nonuniform magnetic field.

In the absence of a temperature gradient between the surfaces the velocity of MHD flow has the profile

$$\frac{v}{\alpha} = \frac{M}{2\operatorname{sh} M}(\operatorname{ch} M - \operatorname{ch} Mx),$$

reflected in Fig. 3. As is seen, its magnitude grows with an increase in M. In the limiting cases for M $\ll$ 1

$$\frac{v}{\alpha} \simeq \frac{(1-x^2)}{2} M^2,$$

while in strong fields (M $\gg$ 1) the velocity depends markedly on M:

$$\frac{v}{\alpha} \simeq \frac{(1-x^4)}{2} M.$$

In the formation of a velocity profile in the region of intermediate values of M the following picture is observed. For Hartman numbers M < 3 the velocity profile has a characteristic property: the velocities near the walls are low, while they are large near the axis. It is clearly seen on the graph that the region of the axis for M = 1 occupies the central part of the channel −0.4 < x < 0.4, while it contracts with an increase in M. At M = 2 it occupies the region −0.2 < x < 0.2, and at M = 3 this region practically disappears.

It is seen from an analysis of Eqs. (6) and (7) that some displacement of liquid arises under isothermal conditions, produced by the nonuniformity of the external magnetic field, i.e., a "magnetic" pump develops.

We follow how the thermal convection affects this liquid flow. The thermal convection does not exert an effect on the magnitude of the liquid flow rate. The effect of convective flow on the movement of the liquid under the effect of an external magnetic field is expressed in a change in the velocity profile of the flow. For this we construct a dependence of the velocity on x in the form:

$$\frac{v}{\alpha} = \beta\,\frac{x\,\mathrm{sh}\,M - \mathrm{sh}\,Mx}{M^2\,\mathrm{sh}\,M} + \frac{M}{2\,\mathrm{sh}\,M}(\mathrm{ch}\,M - \mathrm{ch}\,Mx), \quad \beta = \frac{\mathrm{Gr}}{\alpha}.$$

The effect of free convection on the MHD flow of the liquid for M = 2 and M = 4 is indicated in Fig. 4a, b.

As seen from Fig. 4, free convection exerts a considerable effect on the velocity profile of flow, right up to the appearance in the channel of downward movement of the liquid. For M = 2 the downward flow appears at $\beta = 10$, while for M = 4, as one should expect, downward flow appears in the channel at the higher $\beta = 60$.

Thus, one can draw the conclusion that the stronger the external magnetic field, i.e., the larger the Hartman number M, the weaker is the effect of free convection on the liquid flow arising from the effect of the external nonuniform magnetic field.

We trace whether the generation of MHD flow of the liquid makes a contribution to the vertical convective heat flux:

$$Q_M = \int_{-1}^{+1} v(x)\,\theta(x)\,dx.$$

It is seen from Eq. (4) for the velocity that the second term, produced by the nonuniformity of the magnetic field, does not make a contribution to the convective heat flux, since after integration we obtain an even function mutually compensating within the substitution limits. Thus, the vertical convective heat flux per unit length in the direction of the y axis is:

$$\frac{Q_M}{Q_0} = \frac{45}{M^2}\left(\frac{1}{3} - \frac{\mathrm{cth}\,M}{M} + \frac{1}{M^2}\right),$$

where $Q_0 = 2\mathrm{Gr}/45$ is the heat flux in the absence of a magnetic field. It coincides with the heat flux arising in a vertical channel in an external uniform transverse field [9]. And this should be expected since the developing MHD flow exists under isothermal conditions and consequently does not transfer heat.

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NOTATION

a is the half width of the channel;
$2\theta_1$ is the constant temperature drop between the channel walls;
ν is the kinematic viscosity;
η is the dynamic viscosity;
ρ is the density of the liquid;
μ is the magnetic permeability of the liquid;
σ is the specific conductivity of the liquid;
H is the magnetic field strength;
P is the hydrostatic pressure;
$\varkappa$ is the coefficient of thermal diffusivity;
β is the coefficient of volumetric expansion.

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